Solar Neutrino Zenith Angle Distribution and Uncertainty in Earth Matter Density

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Abstract

We investigated the error in zenith angle distribution for the charged current events of solar neutrino that originated from uncertainty of earth electron density. Basing on a statistical formulation with 5% uncertainty and a proper uncertainty length, we found the error is notable for a correlation $[N]_5/[N]_2,\ [N]_2/[N]_3$.

Forthcoming results from Sudbury Neutrino Observatory (SNO) [1] will include measurements of the day-night asymmetry (A_{DN})[2, 3, 5, 4], which would be a proof of the matter conversion solution of the solar neutrino problem. Furthermore, the zenith angle distribution of events during night is expected to give some insight to distinguish different MSW solutions, LMA, LOW and SMA[6].

Since it is the interaction of neutrinos on the Earth-matter electrons that regenerates ν_e flux, the uncertainty in Earth-matter density and chemical component can be the main error in A_{DN} and the zenith angle distribution. Furthermore, as implicated from the smallness of A_{DN}, the zenith angle distribution is also very delicate, so it necessitates more precise estimation for the errors.

In this paper, we will estimate the error that originated from the uncertainty of Earth electron density(EeD), following the suggestion and formulation established in our previous work[7] and [8] to quantify the density uncertainty for neutrino oscillations. There we have stressed, the variations and uncertainties in Earth density models are defined not only in a relative amplitude, $\delta N_e/N_e$ e.g. a few percentage, but also with some spatial scales δx limited by geophysics experiments and inverting calculations. In general these scale δx are not much larger than the neutrino oscillation length, e.g., of the favored LMA solution. Then the effect of uncertainty might arise beyond linear order and is possible to cause notable errors in zenith angle distributions in solar neutrino experiments.

For simplicity we employ a two-neutrino mixing model, while the neutrino can be treated as a incoherent mixture of two mass eigenstates, as discussed in[5, 9]. During day time the ν_e survival probability is,

$$P^D = P_1 \cos^2\theta + (1 - P_1)\sin^2\theta \tag{1}$$

where we have used a vacuum mixing angle,

$$\nu_1 = \cos\theta\ \nu_e - \sin\theta\ \nu_\mu\ ,\ \nu_2 = \sin\theta\ \nu_e + \cos\theta\ \nu_\mu \tag{2}$$

and P_1 is the probability of the $\nu_e \to \nu_1$ conversion inside the sun[6, 10]. During night time, the presence of earth matter lead to a ν_e a regeneration which is zenith angle dependent,

$$P^N = P_1 + (1 - 2P_1)P_{2e} = P^D - 2X f_{reg}\ , \tag{3}$$

where P_{2e} is the probability of the $\nu_2 \to \nu_e$ conversion inside the Earth, $X = P_1 - 1/2$, and

$$f_{reg}(\theta_z) \equiv P_{2e}(earth\ matter) - P_{2e}(vacuum) \tag{4}$$

is the regeneration factor which will vanish in the absence of the Earth matter. Integrated over the zenith angle, the regeneration factor is prop to the day-night asymmetry,

$$A_{DN} = \frac{2X\bar{f}_{reg}}{0.5 + (\cos 2\theta - \bar{f}_{reg})X} \tag{5}$$

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$$P_{2e}(E_\nu,\theta_z) = \left| \cos\theta \lfloor \mathcal{T}exp\lfloor -i\int_0^{D\cos\theta_z} H[N_e^{\theta_z}(x)]dx \rfloor \rfloor_{ee} + \sin\theta \lfloor \mathcal{T}exp\lfloor -i\int_0^{D\cos\theta_z} H[N_e^{\theta_z}(x)]dx \rfloor \rfloor_{e\mu} \right|^2 \quad (6)$$

where $D = 12742$ is the diameter of Earth in kilometer and $H[N_e^{\theta_z}(x)]$ is the Hamilton for the trajectory at zenith angle θ_z,

$$H[N_e^{\theta_z}(x)] = \frac{\Delta m^2}{4E_\nu}\begin{pmatrix} 2\sin^2\theta & \sin 2\theta \\ \sin 2\theta & 2\cos^2\theta \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_F N_e^{\theta_z}(x) & 0 \\ 0 & 0 \end{pmatrix} \quad (7)$$

and $N_e^{\theta_z}(x)$ is the relevant EeD on this trajectory. For each zenith angle there will be a unique solution for regeneration factor $f_{reg} = P_{2e} - \sin^2\theta$ when the density is completely determined. As shown in Fig.1.(a), the regeneration factor has a periodic shape, which give a rough estimation for the neutrino oscillation length in Earth matter. Since different MSW solution predict different oscillation length, after an integration over neutrino energy, the shapes is expected to lead to the features for MSW solutions.

However the available electron density is known only to certain precision. [12, 8, 11]. The uncertainties of Earth matter density means a an error in $\neq_e$ survival probability during night time, and we now estimate it using the procedure developed in [7]. In a neighborhood of any point x, the realistic matter density accesses a value $N_e(x)$ with a probability $F[N_e(x), x][\mathcal{D}N_e]$, where

$$F[N_e(x), x] = \frac{1}{N_e(x)\sqrt{2\pi}s(x)} exp\{-\ln^2\lfloor\ N_e(x)/N_0(x)\ \rfloor/\lceil 2s^2(x)\rceil - [N_e'(x)]^2/\lceil 2q(x)\rceil\}$$
$$s(x) = \sqrt{\ln\lceil 1 + p^2(x)\ \rceil}, \quad N_0(x) = \hat{N}_e(x)exp\lfloor -s^2(x)/2\rfloor \quad (8)$$

For small uncertainty it is a Gaussian distribution around $\hat{N}_e(x)$, the prediction of a density model. Here we have also defined the precision function

$$p(x) = \sigma(x)/\hat{N}_e(x)\ , \quad (9)$$

a relative quantity to characterize the precision of electron density. The second term in the exponent is introduced for the smoothness of density profiles. Except for some geophysical discontinuity, the profiles with a large gradient $N_e'(x)$ will be suppressed with a proper $q(x)$ for dimension. Without loosing generality, we set this term to vanish as a first estimation. Mathematically, $\hat{N}_e(x)$ and $\sigma(x)$ are defined in a infinitely small neighborhood of x. In physics, there exists a minimal volume $\propto \delta x$ for the neighborhood when we cite specific amplitude for $\sigma(x)$. We keep this electron density uncertainty scale δx different from that scale of density change $l_\rho \equiv \rho/\frac{d\rho}{dx}$ which characterizes the flatness (adiabaticity) of density profile. They are both important for neutrino oscillation in matter, while, the effect of l_ρ can be included by performing exact numerical calculations with realistic density models, e.g. PREM[13], but the effect of uncertainty scale can not be decreased unless more precise density models is available. Furthermore, in the case where δx is comparable to the neutrino oscillation length in Earth matter, one has to be careful in estimating the errors for oscillation probability. In the language of [11] (Eq.2 there), this case means the rising of the nonlinear term of $\delta x \delta N_e$. Averaging the uncertainties we have,

$$f_{reg}(\theta_z) + \sin^2\theta =< P_{2e}(E_\nu,\theta_z) >= \int P_{2e}(\theta_z)F[N_e^{\theta_z}(x)][\mathcal{D}N_e^{\theta_z}]\ ,$$
$$\delta f_{reg} \quad = \quad \delta P_{ee}(E_\nu,\theta_z) \equiv \sqrt{< P_{ee}^2(E_\nu,\theta_z) > - < P_{ee}^2(E_\nu,\theta_z) >^2} \quad (10)$$

The practical evaluation of above path integrals involves a descretizing of the trajectories of neutrino, and we refer to our previous work[7] for the details. Given [12], we take a rough 100 Km bin size for the radius of earth for 5% uncertainty of PREM, and get a uncertainty scale by projecting it onto the neutrino trajectory. Here we only give a plot of δf_{reg} around f_{reg}, with the global fitted oscillation parameters[14], *i.e.* ,

$$LMA\ :\ \Delta m_{12} = 3.7\ 10^{-5}, \tan^2\theta = 3.7$$
$$LOW\ :\ \Delta m_{12} = 1.0\ 10^{-7}, \tan^2\theta = 7.6 \quad (11)$$

In Fig.1.(b), we can see LMA suffers a larger error. For LOW the error is rough two percentage and for SMA it is smaller, so we neglected them in the figure. Integrated over zenith angle distribution, it means a is still smaller than the difference between A_{DN}^{LMA} and A_{DN}^{LOW}

To see the effect in solar neutrino observation, we explore the error in the rate of the charged current events in the night time.

Following [6] we define the normalized rate of the charged current events,

$$[CC](\theta_z) \equiv N_{CC}/N_{CC}^{SSM} = \int_{E_\nu^0} dE_\nu \Phi(E_\nu) P^N(E_\nu, \theta_z) \sigma_{CC}(E_\nu) / \int_{E_\nu^0} dE_\nu \Phi(E_\nu)\ Unit\ \sigma_{CC}(E_\nu) \quad (12)$$

where $\Phi(E_\nu)$ contains both the neutrino flux from Boron decay and $He + proton$ chain in the sun[4, 15], and σ_{CC} is the charged current cross section of neutrino on Deuteron. Since the uncertainty from recoil electron kinetics might be canceled in $[CC]$ as a ratio, we have employed a quick access through an interpolation[16] with a starting point $E_\nu^0 - 1.447$ just at the threshold energy of electron $E_{th} = 5MeV$. At the same time, we have neglected all the other possible error. We plotted the zenith angle distribution for above charged events rate, as Fig.2. It can be taken as a check of our calculation, which repeats the fact that, SNO charged current data lies just in the middle between the best fit of LMA and LOW. As a numerical illustration, we also follow the binning method in [6]. Considering of the charged current events, we take the fifth bin $.83 \sim .92$ like SNO. In Fig.3.(a) and Fig.3.(b), we can recognize the relations,

$$\begin{aligned} LMA &: [N]_1 < [N]_2 \leq [N]_3 \leq [N]_4 \leq [N]_5 \\ LOW &: [N]_2 \geq [N]_4 > [N]_1 \sim [N]_3 > [D] \end{aligned} \quad (13)$$

More quantitatively, it reads, $[N]_5/[N]_2 = 0.999 \approx 1, [N]_2/[N]_3 = 0.995 \approx 1$ for LMA while $[N]_5/[N]_2 = 0.982, [N]_2/[N]_3 = 1.053$ for LOW. Then one may have a discrimination between LMA and LOW. The solution of SMA can be considered as another LOW but with different feature, for simplicity we will not repeat that similar calculation. Anyway, for such delicate features, we now exam the error from density uncertainty through,

$$\delta[CC] \propto \int_{E_\nu^0} dE_\nu \Phi(E_\nu)(-2X\delta f_{reg})\sigma_{CC}(E_\nu) \quad (14)$$

and re-plot the charged events rate in the night with error-bars. To avoid multi-fold integration which is computer time consuming, we investigated δf_{reg} at neutrino energy of 8, 10, 11, 12 Mev and found they are close to each other. To be conservative we have ever used the maximal value for δf_{reg}.

Using Eq.(11) and 5% precision of PREM at uncertainty scale of 100 Km, we see that the errors originated from EeD rises larger and larger with the increasing of zenith angle for LMA in Fig.4.(a).

For quantity, we also average it in bins. From Fig.4.(b), we can see that the errors lead to $([N]_5 - \delta[N]_5)/[N]_2 \approx 0.935$ while $([N]_2 + \delta[N]_2)/([N]_3 - \delta[N]_3) \approx 1.043$. Having noticed the LMA sheet in the correlation figures mainly stretched along A_{DN} direction (*e.g.* Fig.13∼ Fig.16 in [6]), we defined and investigated a correlation between $[N]_5/[N]_2$ and $[N]_2/[N]_3$ as in our Fig.5. Then the point (1, 1) for LMA is swollen into a rectangle whose side is very close to the point (0.982, 1.053) for LOW. Small error bars for LOW has been ignored, since it will lead to more overlap. If precision of PREM is 10% in the same uncertainty scale, the situation will change much remarkable.

Naturally, modern Earth's density models with higher precision will be useful to shrink such an error. In Fig.6, we re-plot the content of Fig.4.b, but with density model AK135[17]. Its precision is said to be $1 \sim 2\%$, and we can infer natural the uncertainty scale as rough 50 Km, since the model was presented in a data table. Now the error from 2% uncertainty is not so large , *e.g.* in Fig.6.(a) one can find $([N]_5 - \delta[N]_5)/([N]_2 + \delta[N]_2) \approx 1.033$ while $([N]_2 + \delta[N]_2)/([N]_3 - \delta[N]_3) \approx 1.017$. From Fig.6.(b), one finds the distance from LMA to LOW is restored relatively, anyway, it implicates that the errors can be decrease by employing more precise geophysics data.

In summary, we estimated the errors in zenith angle distribution that originated from electron density uncertainty for charged current solar neutrino event rate in the night time. We noted the uncertainty scale of density is not much far from the neutrino oscillation length in Earth matter. The fluctuation is not so large in the LOW and SMA case, however, it will bring notable error to LMA. Although it is only an estimation at specific parameters, qualitatively, the observation of zenith angle distribution might necessitates more precise knowledge on electron density of Earth.

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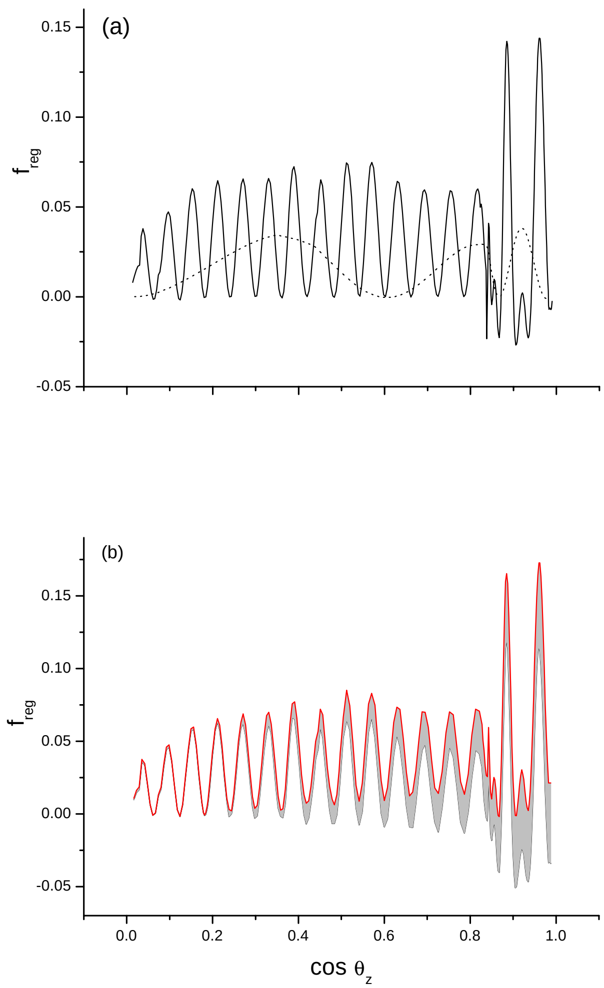

Figure 1: Regeneration factor via zenith angle for neutrino at 11 Mev through Earth matter with PREM. Parameters in Eq.(11) have been used. (a). The solid line is for LMA while the dotted line for LOW (b). the errorbars is attached from 5% uncertainty in matter density (PREM) . The fluctuation in LOW case is smaller than the LMA case, so we have neglected it.

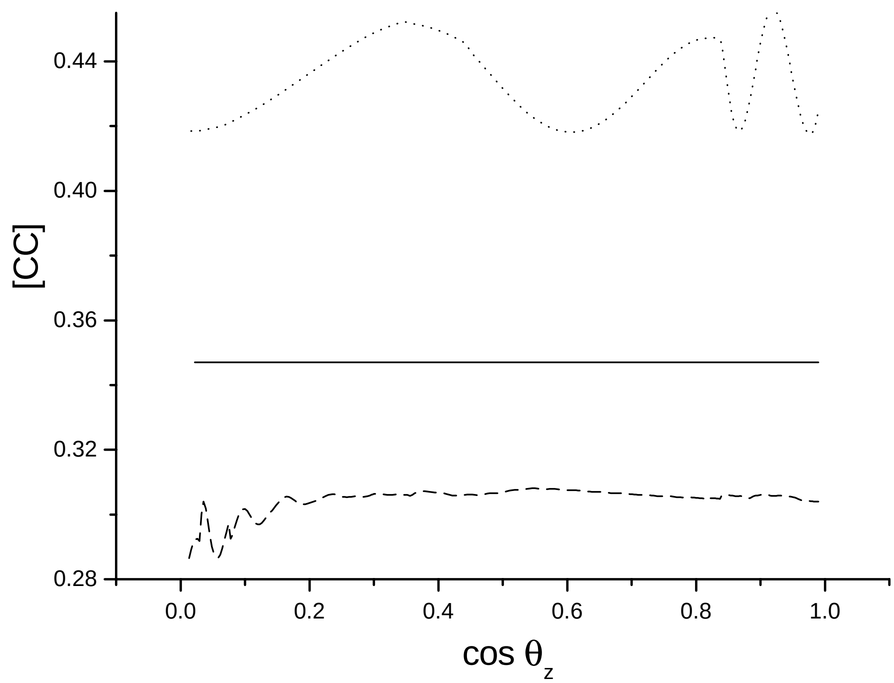

Figure 2: charged events rate via zenith angle. The dashed line above is for LOW and the dot line below for LMA. The solid straight line in the middle is SNO observation.

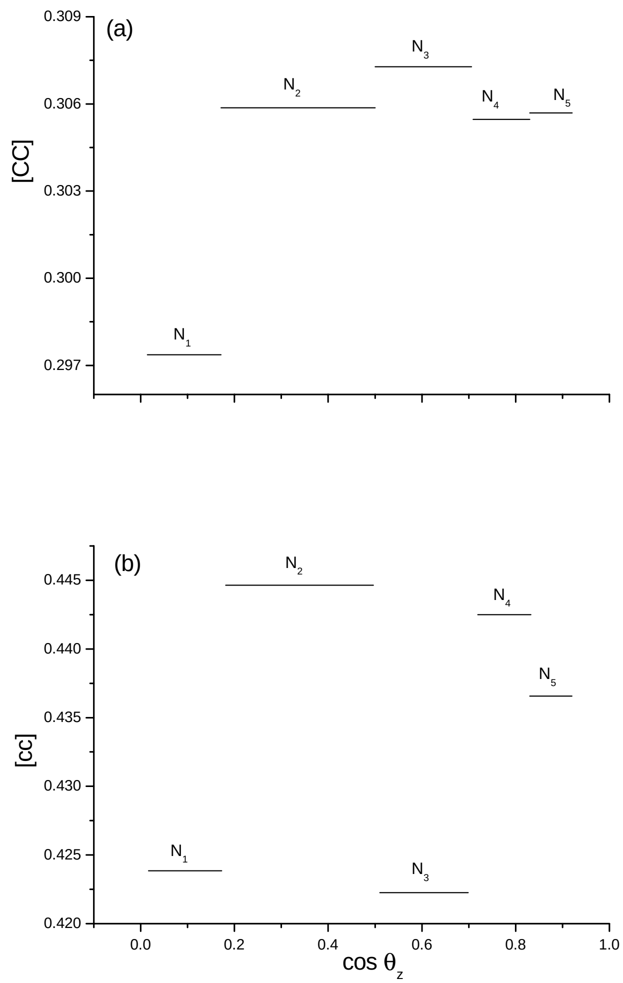

Figure 3: Same as Fig.2, but averaged in five bins of zenith angle. (a) for LMA , the dashed line in Fig.2, (b) for LOW , the dotted line in Fig.2.

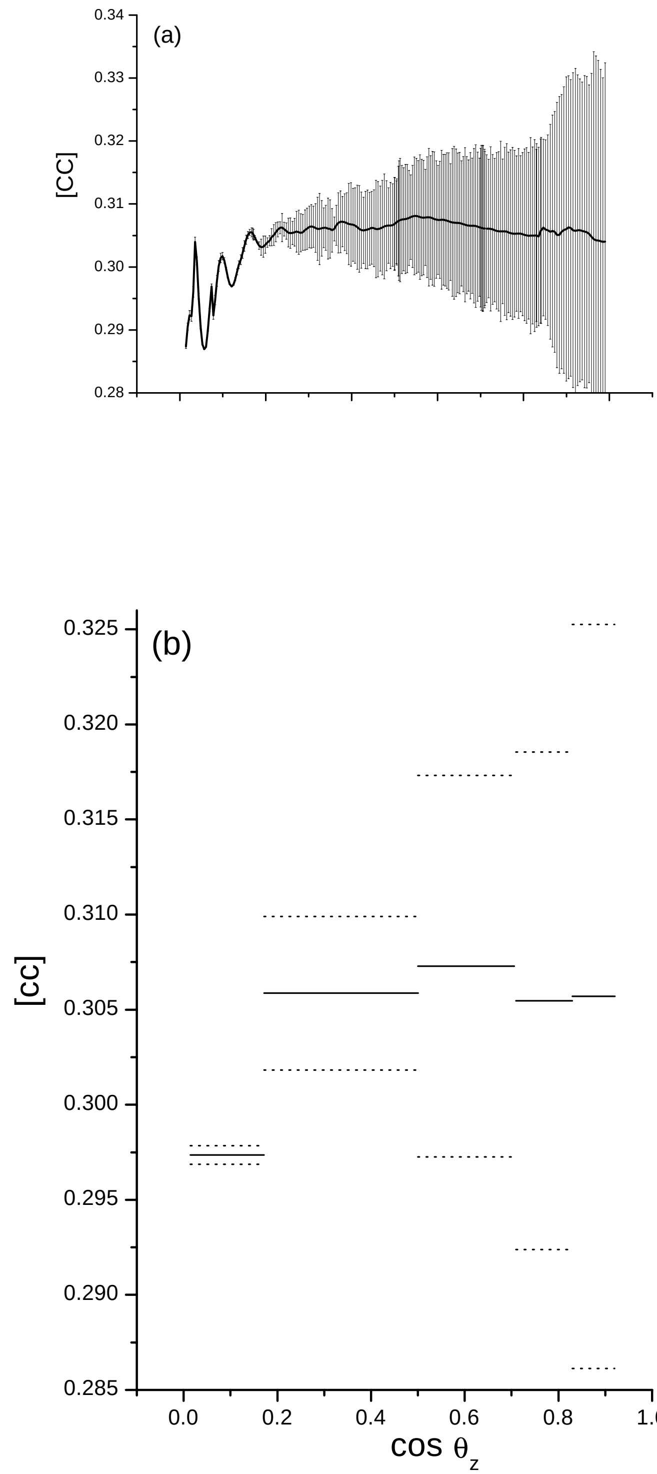

Figure 4: Error in charged current event rate via zenith angle. (a) Error-bars attached on the dashed line of Fig.2. (b) Same as (a) , but is averaged in five bins.

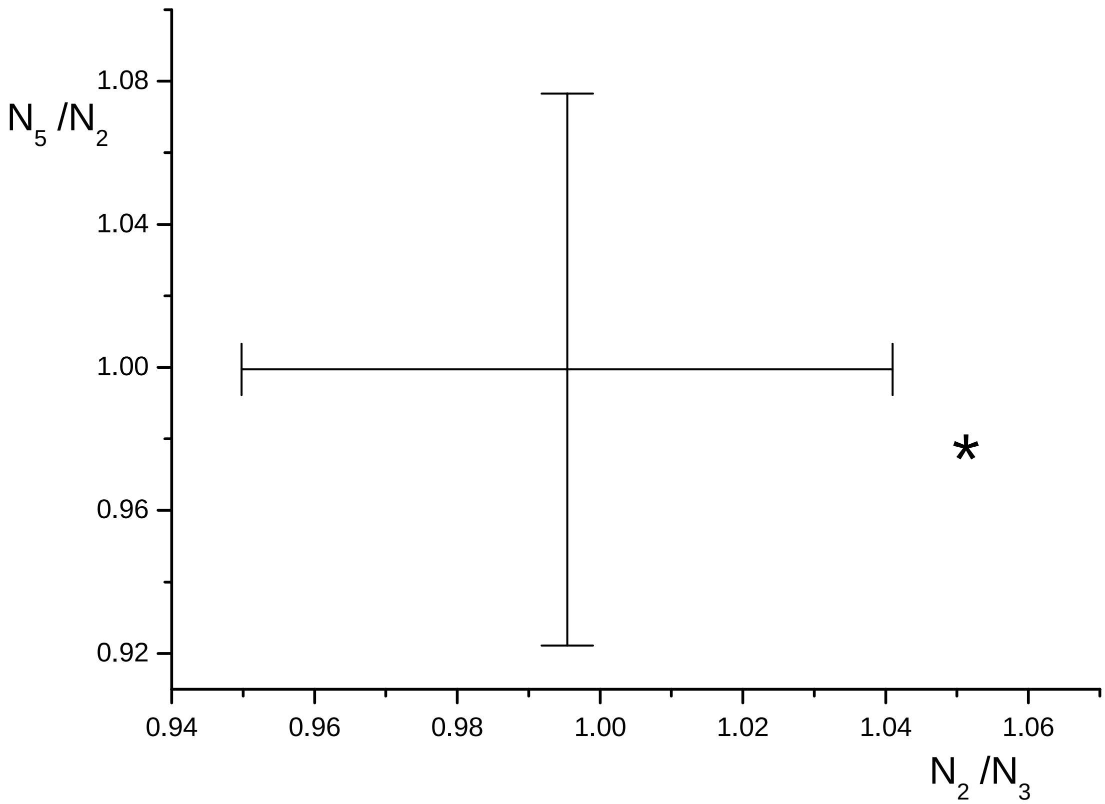

Figure 5: A combination of Fig.3 and Fig.4.(b), the corellation between the ratio of event in important bins, N_5/N_2 via N_2/N_3 . The center of of the cross is the LMA point, while the start is for LOW . The error bars originated from the uncertainty of electron density .

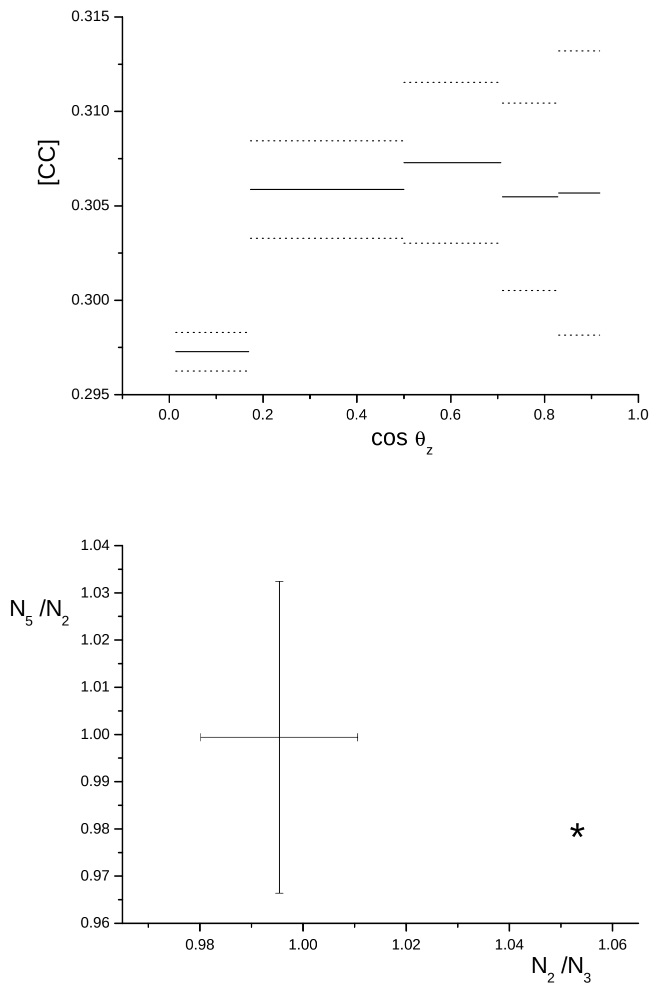

Figure 6: (a) is same as Fig.4.(b), but with 2% electron density uncertainty in AK135 model. (b) Same as Fig.5. but from AK135. The error bar shrins much.